SAR principles

E. Colin Koeniguer, elise.koeniguer@onera.fr

Outline

1. Introduction to SAR
    - Introductions
    - Resolutions

Outline

Outline

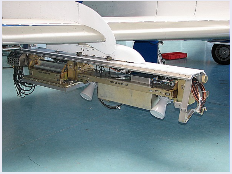
1. Introduction to SAR
    - Introductions
    - Resolutions
2. Understanding SAR: different points of view
    - Geometric point of view
    - The PSF point of view
    - The signal processing point of view
    - Why different SAR algorithms?
3. Example of a SAR algorithm: the RMA
    - Modelling of the mathematical problem to solve
    - The different steps of the algorithm

What is SAR?

Definition

- Synthetic Aperture Radar (SAR) is an active remote sensing technology that uses microwave energy to illuminate the surface.
- The system records the elapsed time and energy of the return pulse received by the antenna.
- These instruments can collect data about the Earth's surface day or night, regardless of cloud cover.

Example of airborne system



Why doing SAR?

Specificities

- Because Radars opperate at relatively long wavelengths, they are not (or only very little) affected by scattering in the atmosphere, thus they can ”see” through clouds.
- The reflected signal depends (among other factors) on the surface roughness, which provides important additional information not directly available from other observations.
- Radar waves used to visualize objects because of their ability to penetrate a range of materials

SAR concept

Principle

The Radar is an active remote sensing system. Short pulses of EM radiation are sent out and the reflected signal is detected.

Range resolution

travel time t given by $\tau = \frac{2x}{c}$, x is the distance travelled The range resolution is then given by the length of pulse $\Delta\tau$:

$$\delta_x = \frac{\Delta\tau c}{2}$$

If frequency modulation is used with a bandwidth B then

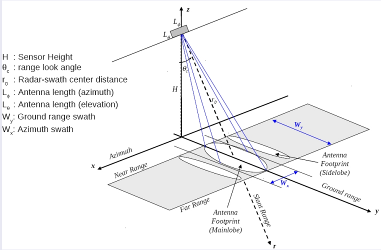
$$\delta_x = \frac{c}{2B}$$

SAR concept

Azimuth resolution

- Azimuth Resolution of image improves as aperture size increases.
- Unfortunately, increasing aperture size (antenna length) may simply be impractical (antenna lengths in kilometers)

beam resolution



Example

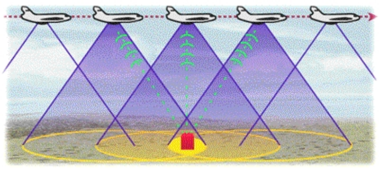
Uniform aperture antenna has a lobe with an aperture angle $\phi = \frac{\lambda}{L}$ where L is the antenna dimension.
$L = 10\text{m},\ \lambda = 20\text{cm}$
$\Longrightarrow \phi = 0.02 rad = 1.145^\circ$ for
H=800km, $\theta_0 = 20^\circ \Longrightarrow R = 900 km$
$\delta_y = 2R \tan\frac{\phi}{2} \approx R\phi = 18 km$!!

SAR concept

Goal

gain the advantages of a large aperture radar by using a smaller, traveling aperture

Azimuth processing



Geometric properties of a SAR acquisition

SAR geometric principles

- Points of the ground are differentiated by their distance
- Points of the ground are differentiated by their doppler

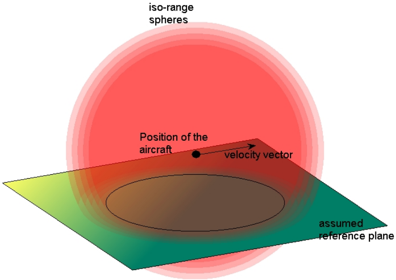
Separation according to distance

- What are the set of points located at a given distance of the antenna ?
- What are the intersection of this set with the ground if it is assumed to be an horizontal plane ?

Isorange

Geometric considerations

- Iso range surfaces are spheres
- Intersection of a sphere with a plane : circles



Geometric properties of a SAR acquisition

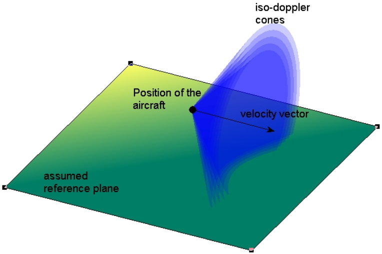
Separation according to doppler

- What are the set of points seen by a given velocity relative to the antenna ?
- What are the intersection of this set with the ground if it is assumed to be an horizontal plane ?

Isodoppler

Geometric considerations

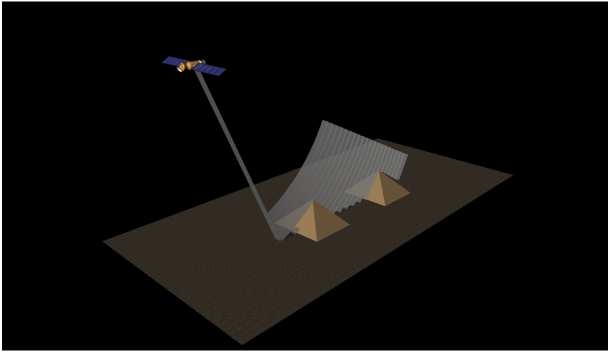
- Iso doppler surfaces are cones
- Intersection of a cone with a plane: quadric curve



Geometric properties of a SAR acquisition

2D projection of 3D data

The range cell is the projection of a range voxel on the ground

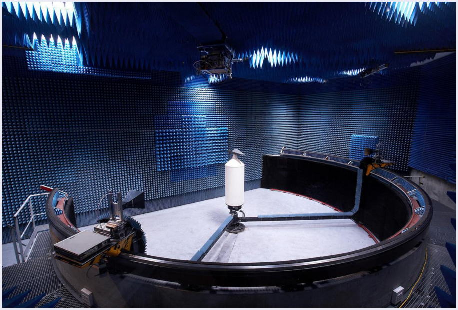


Exemple of Mapping on the ground

On a horizontal plane

Exemple of Indoor measurement

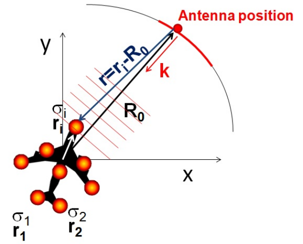
BABI, a anechoic chamber allowing imaging



Antennas can moove on a circular rail.

Imaging from indoor measurement (1/2)

Modelling of the signal



Modelling of the signal

$$H(\mathbf{k}) = \int exp(j\mathbf{kr})\sigma_i dS_i exp(j\mathbf{kr})$$

$$H(\mathbf{k}) = \int \sigma_i dS_i exp(j2\mathbf{kr_i}) exp(-j2\mathbf{kR_0})$$

$$H(\mathbf{k}) exp(j2\mathbf{kR_0}) = \int \sigma_i dS_i exp(j2\mathbf{kr_i})$$

Imaging from indoor measurement (2/2)

Modelling of the signal

$$H(\mathbf{k})exp(j2\mathbf{k}\mathbf{R_0}) = \int dS_i\sigma_i exp(j2\mathbf{k}\mathbf{r_i})$$

with $\mathbf{k} = \begin{pmatrix} k\cos\theta \\ k\sin\theta \end{pmatrix}$ and $\mathbf{r_i} = \begin{pmatrix} x \\ y \end{pmatrix}$, $\sigma_i = \sigma(x,y)$ et $dS_i = dx\ dy$

$$H(\mathbf{k})exp(j2\mathbf{k}\mathbf{R_0}) = \int dxdy\sigma(x,y)exp(j2k_x\cos\theta x)exp(j2k_y\sin\theta y)$$

Let $K_x = 2k\cos\theta$ and $K_y = 2k\sin\theta$

Final signal

$$exp(j2\mathbf{k}\mathbf{R_0})H(\mathbf{k}) = \int dxdy\sigma(x,y)exp(j(K_x x + K_y y))$$

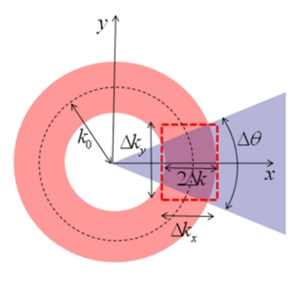
$$\sigma(x,y) = TF^{-1}(exp(j2\mathbf{k}\mathbf{R_0})H(\mathbf{k}))$$

Computation

Methods

Approximate the frequency support by a rectangular box.

Resolutions



Resolutions

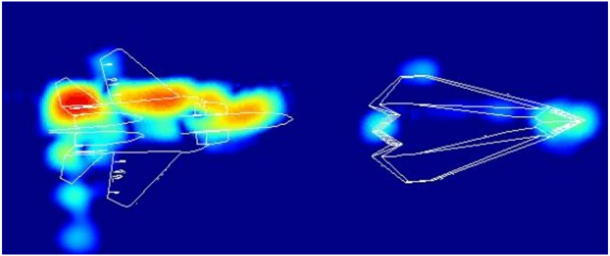
$$\Delta k_x = 2\Delta k = 4\pi\frac{\Delta f}{c}$$

$$\Delta k_y = 2k_0\Delta\theta$$

$$\delta_x = \frac{2\pi}{\Delta K_x}, \delta y = \frac{2\pi}{\Delta K_y}$$

$$\delta_x = \frac{c}{2\Delta f}, \delta y = \frac{c}{2f_0}\frac{1}{\Delta\theta}$$

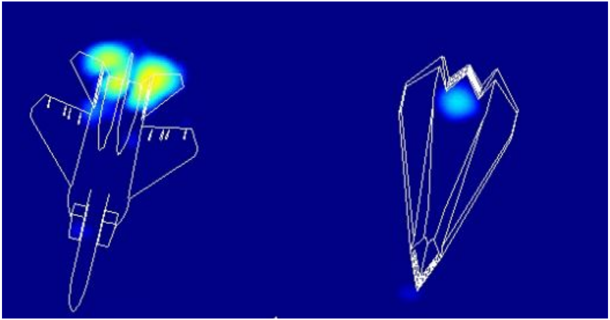
Examples of images from indoor measurement



Questions

- Where is the stealth aircraft?
- On which parameters depends the resulting image?

Examples of images from indoor measurement



Questions

- Where is the radar?
- Order of magnitude of the resolutions?

The case of SAR acquisition

Goal

Determine $\sigma(x, y)$ for each target

How?

- Range imaging
- Cross Range imaging

Computation

Why different SAR algorithms?

- Computational cost
- Adaptability to large bandwidths or large swath
- Motion compensation

Which different SAR algorithms?

- Backprojection or temporal correlation
- RDA Range Doppler algorithms or Polar Algorithm
- RMA Range Migration Algorithm or (ω, k) Algorithm

Comparisons of different SAR algorithms

Temporal correlation, backprojection

- exact geometry
- time consuming ⟶ backprojection: one interpolation

RDA or PFA, CSA : frequential domain

- the most used on satellite platforms
- only one interpolation
- not adapted to very large bandwidths
- CSA version adapted the the chirp signal

RMA : dual frequential domain

- adapted to very large bandwidths
- Motion compensation not adapted to circular trajectories

The initial problem to solve

Modelling of the signal

$$s(t,u) = \iint dxdyf(x,y)s_0(t - \frac{2}{c}d(x,y))$$

The goal

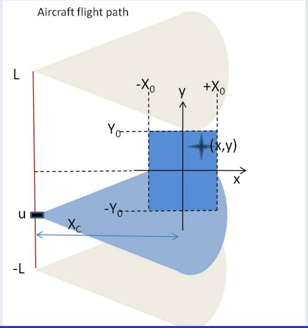
How to deduce $f(x,y)$?

A particular case

linear trajectory, non squinted, stripmap mode

$$d(x,y) = \sqrt{(X_c + x)^2 + (u-y)^2})$$

Geometric parameters



First Step

Notations

Let $X = (X_c - x)$ and $Y = (u - y)$. Note that $X > 0$

Fourier transform along t

$$f_s(f,u) = \iint dxdyf(x,y)\,TF_{t\mapsto\omega}\left(s_0\left(t - \frac{2}{c}\sqrt{X^2+Y^2} + t_0\right)\right).$$

$$s(t-a) \underset{FFTt\mapsto\omega}{\longrightarrow} S(\omega)exp(-i\omega a)$$

$$f_s(f,u) = \iint dxdyf(x,y)S_0(f)\exp\left(-j2\pi f\frac{2}{c}\sqrt{X^2+Y^2}\right)\exp\left(j2\pi f\frac{2}{c}X_c\right).$$

Second Step

Matched filtering

We want simply $\frac{f_s(f,u)}{S_0(f)}$ but $S_0(f)$ can have zero values. We do instead match filtering:

$$f_{sm}(f,u) = f_s(f,u) S_0(f)^*$$

Result

$$f_{sm}(f,u) = |S_0(f)|^2 \iint dx\, dy\, f(x,y) \exp\left(-j2\pi f \frac{2}{c}\sqrt{X^2+Y^2}\right) e^{j2\pi f \frac{2}{c} X_c}.$$

if $|S_0(f)|^2 = 1$ over the bandwidth:

$$f_{sm}(f,u) = \iint dx\ dy\ f(x,y)\ \exp\left(-j2\pi f \frac{2}{c}\sqrt{X^2+Y^2}\right) e^{j2\pi f \frac{2}{c} X_c}.$$

Third Step

Fourier transform along *u*

FFT transform along *u* axis

$$F_s(f, k_u) = \int f_{sm}(f, u) \exp(-jk_u u) du$$

$$F_s(f, k_u) = \iiint dx\ dy\ du\ f(x, y)\ e^{\left(-j2\pi f \frac{2}{c}\sqrt{X^2+Y^2}\right)} e^{j2\pi f \frac{2}{c} X_c} e^{-jk_u u}.$$

Stationary method for integral over u

$$F_s(k, k_u) = \iint dx\ dy\ f(x, y) e^{-j\frac{\pi}{4}} e^{\left(-j\sqrt{4k^2-k_u^2}(X_c+x)-jk_u y\right)} e^{j2kX_c}.$$

Fourth Step

frequence centering

by multiplying our function by $e^{j\pi/4}e^{j\sqrt{4k^2-k_u^2}X_c}e^{-j2kX_c}$, we obtain:

$$F_{sm}(k,k_u)=\iint dx\ dyf(x,y)\exp\left(-j\sqrt{4k^2-k_u^2}x-jk_uy\right).$$

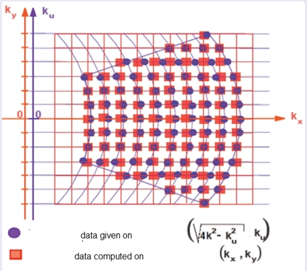
Fifth Step

Stolt interpolation

let $k_x = \sqrt{4k^2 - k_u^2}$ and $k_y = k_u$. Then

$$F_{sm}(k_x, k_y) = \iint dx\ dy\ f(x, y) \exp\left(-jk_x x - jk_y y\right).$$

Schematic representation $F_{sm}(k, k_u) \longrightarrow F_{sm}(k_x, k_y)$



Sixth and last step

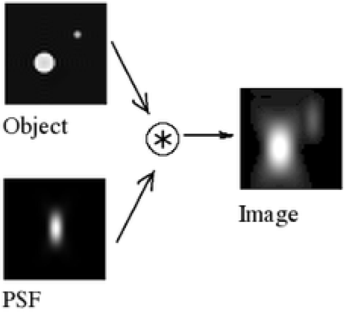
2D FFT

$$f(x, y) = \iint F_{sm}(k_x, k_y) dxdy \exp\left(+jk_x x + jk_y y\right).$$

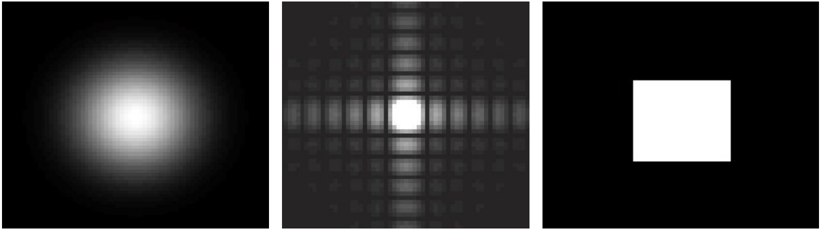
The Point Spread Function

Definition

describes the response of an imaging system to a point source or point object.



The Point Spread Function



Question

- What is the Point Spread Function for a SAR image ?
- Why?

Why is it far more complex?

Geometry is more complex

- squinted modes (middle of trajectory does not correspond to middle of the scene
- 3D mode: the antenna has an elevation
- motion compensation: the trajectory is not exactly linear

Towards more sophisticated configurations

- towards ultra high resolution: very sensible to trajectory
- towards other trajectories: circular
- towards bistatic configurations

To conclude

The only things to keep in mind

- SAR is a 2D image process
- Points are differentiated by their distance and doppler
- Range resolution is obtained like with a classical radar: using compression with a frequency bandwith.
- Azimuth resolution is obtained with the movement of the antenna. ("Synthetic antenna"). It depends also on the central frequency.